Multivariate Analysis

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MedStatStudio

Objectives

- Demonstrate the use of an appropriate statistical test for multivariate hypothesis testing including Chi Square test, t-tests, and the correlation coefficient.
- Understand the role of logistic regression, ANOVA, and linear regression.

Hypothesis Testing

1. Identify parameter of interest
2. Determine null and alternative hypothesis
3. Define the test statistic***
4. State rejection region
5. Calculate test statistic
6. Decide if H_0 will be rejected
7. State conclusion in context

Quiz

MedStatStudio

Methodology of Research in Emergency and Disaster Medicine

Quiz 7

Multivariate Hypothesis Testing

For the following scenarios, indicate which would be the best test statistic.

A. Chi-Square Test
B. Logistic Regression
C. Paired t-test
D. Two Sample t-test
E. ANOVA
F. Wilcoxen Rank Sum Test (Mann Whitney Test)
G. Linear Regression
H. Correlation Coefficient

Triage Accuracy

A researcher is investigating the accuracy of triage code assignment with two triage systems (CTAS and START). 90 patient cases were evaluate by each system. Among the CTAS group, 70 were correct. Among the START group, 65 were correct.

Triage Accuracy

1. Parameter of interest:
   - ???

Triage Accuracy

1. Parameter of interest:
   - P_S and P_C (proportion correct)

Triage Accuracy

1. Parameter of interest:
   - P_S and P_C (proportion correct)
2. Determine null and alternative hypothesis
   - ????

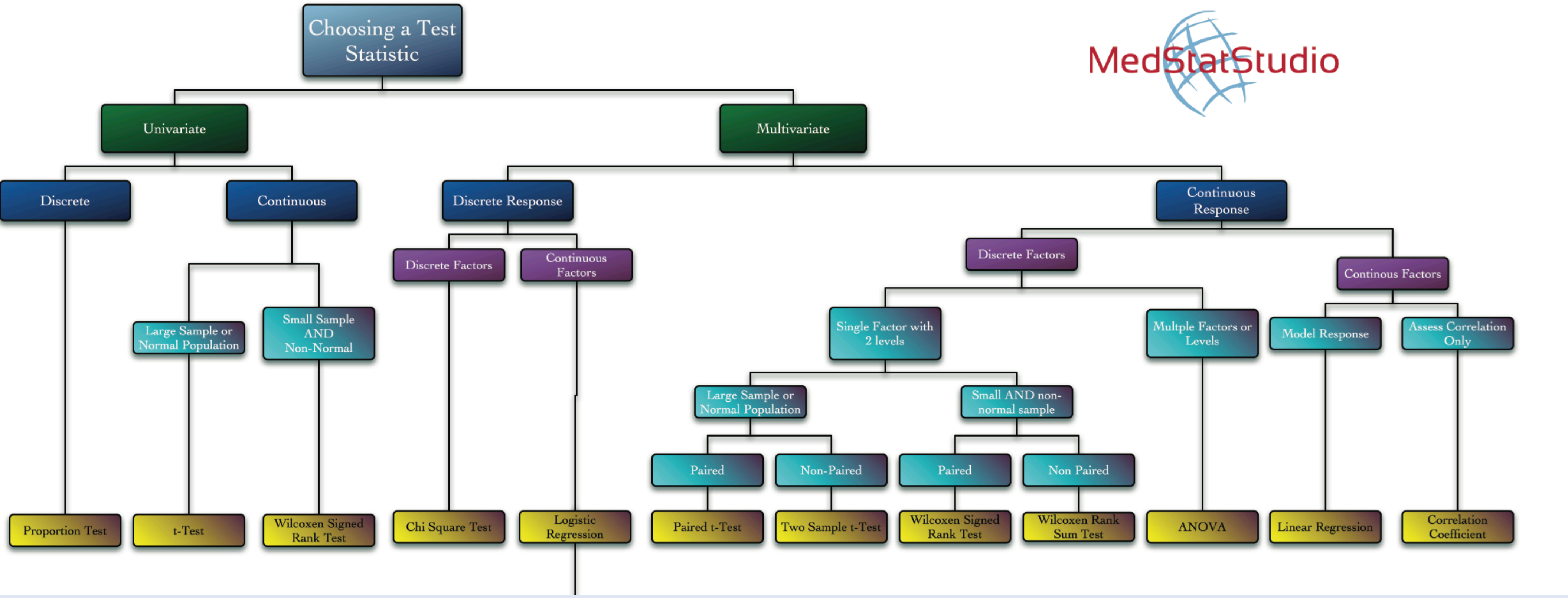
Triage Accuracy

1. Parameter of interest:
   - P_S and P_C (proportion correct)
2. Determine null and alternative hypothesis
   - H_0: $P_S = P_C$
   - H_A: $P_S \neq P_C$

4. State Rejection Region
   - $\alpha = 0.05$
   - Reject if $p < 0.05$

Triage Accuracy

3. Define the test statistic



Triage Accuracy

3. Define the test statistic

Multivariate

↓

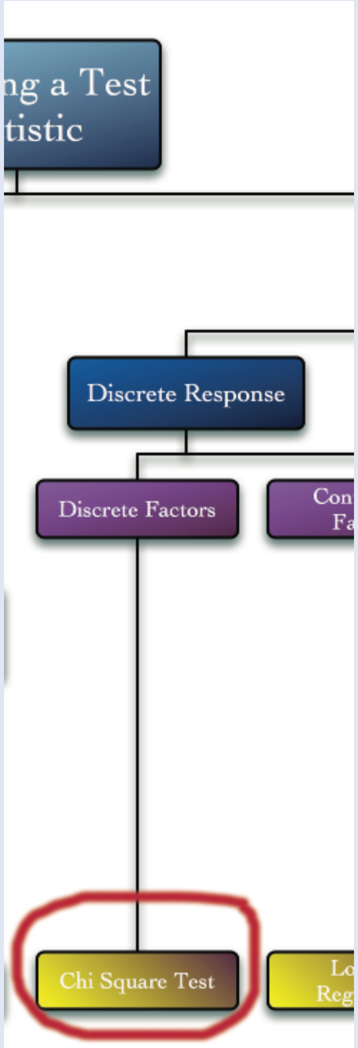
Discrete Response

↓

Discrete Factors

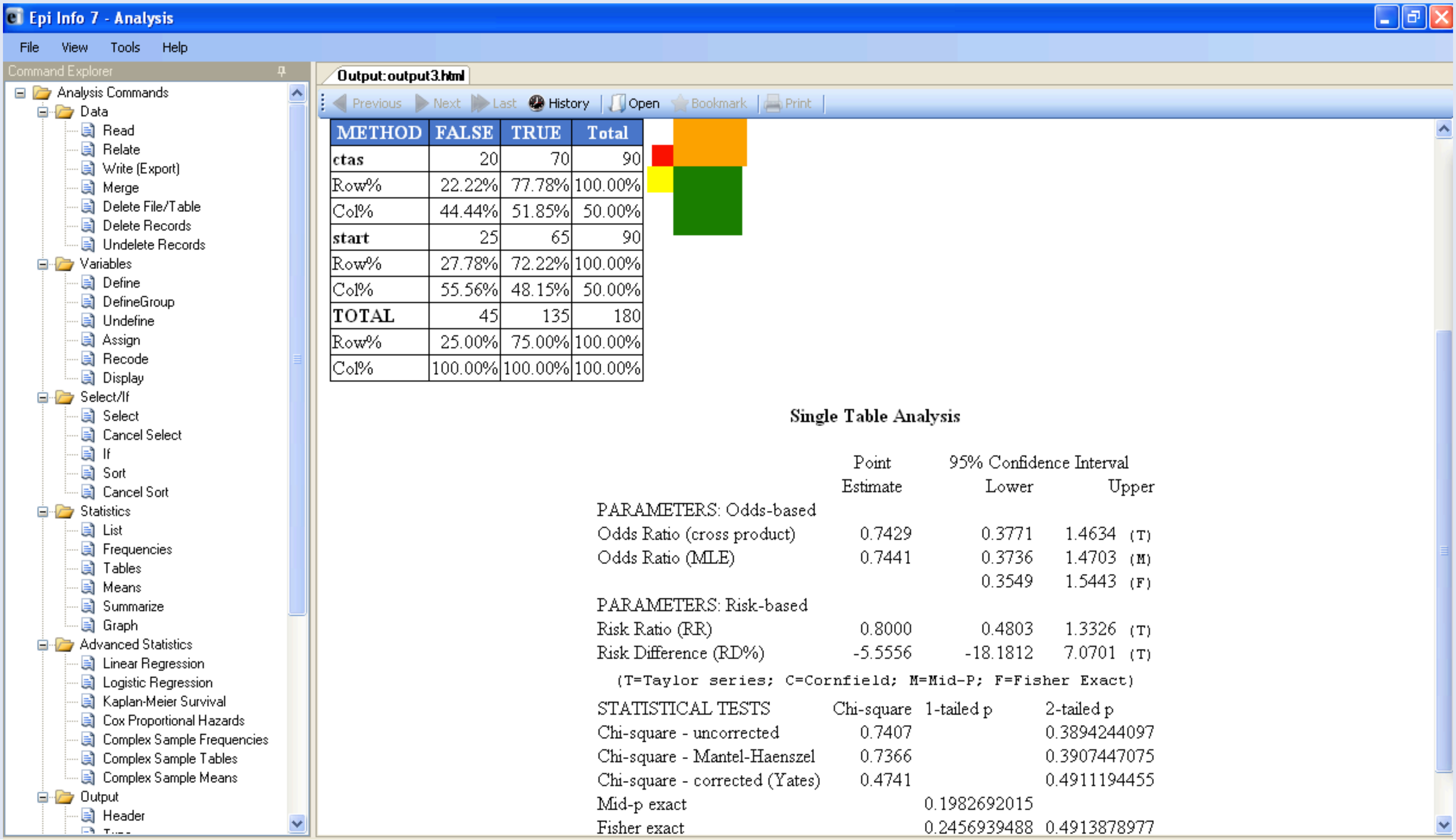
↓

Chi-Square Test



Triage Accuracy

5. Calculate the test statistic



METHOD	FALSE	TRUE	Total
ctas	20	70	90
Row%	22.22%	77.78%	100.00%
Col%	44.44%	51.85%	50.00%
start	25	65	90
Row%	27.78%	72.22%	100.00%
Col%	55.56%	48.15%	50.00%
TOTAL	45	135	180
Row%	25.00%	75.00%	100.00%
Col%	100.00%	100.00%	100.00%

Single Table Analysis

	Point Estimate	95% Confidence Interval Lower	Upper	
PARAMETERS: Odds-based				
Odds Ratio (cross product)	0.7429	0.3771	1.4634	(T)
Odds Ratio (MLE)	0.7441	0.3736	1.4703	(M)
		0.3549	1.5443	(F)
PARAMETERS: Risk-based				
Risk Ratio (RR)	0.8000	0.4803	1.3326	(T)
Risk Difference (RD%)	-5.5556	-18.1812	7.0701	(T)

(T=Taylor series; C=Cornfield; M=Mid-P; F=Fisher Exact)

STATISTICAL TESTS	Chi-square	1-tailed p	2-tailed p
Chi-square - uncorrected	0.7407		0.3894244097
Chi-square - Mantel-Haenszel	0.7366		0.3907447075
Chi-square - corrected (Yates)	0.4741		0.4911194455
Mid-p exact		0.1982692015	
Fisher exact		0.2456939488	0.4913878977

Triage Accuracy

The overall triage accuracy was 77% for the CTAS group and 72% for the START group. The null hypothesis of no difference between triage accuracy was not rejected as there was no significant statistical difference ($p>0.3$).

Admission Prediction

A researcher wishes to develop a prediction model for admission to hospital related to 8 variables:

1. Age
2. Gender
3. CTAS Triage Score
4. Pulse
5. Respiratory Rate
6. GCS
7. Systolic BP
8. Mode of arrival (Ambulance or not)

Admission Prediction

1. Parameter of interest

Admission Prediction

1. Parameter of interest
   - Y=admission yes/no
   - x_i=‘factors’
   - $Y=\beta_0+\beta_1 x_1+\beta_2 \beta x^2+\beta_3 x_3+\beta_4 x_4$…. .

Admission Prediction

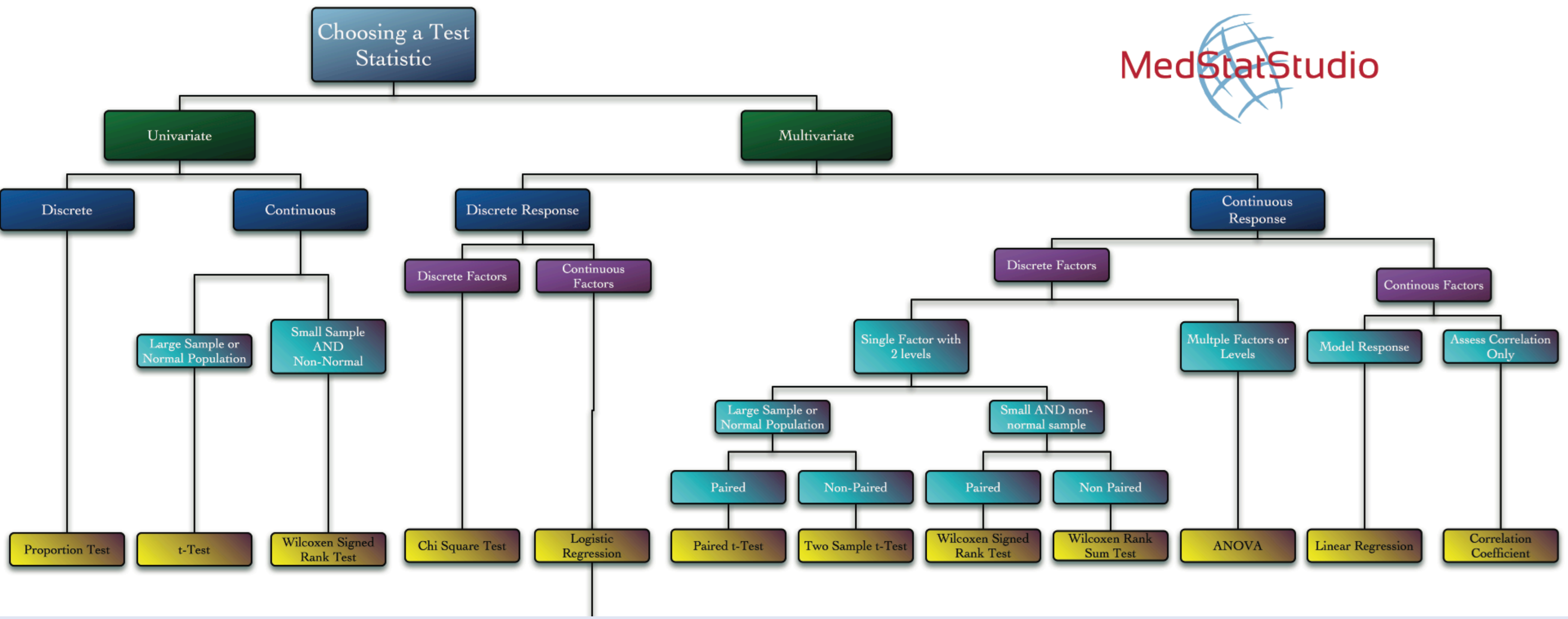
1. Parameter of interest
   - Y=admission yes/no
   - x_i='factors'
   - $Y=\beta_0+\beta_1 x_1+\beta_2 \beta x^2+\beta_3 x_3+\beta_4 x_4$…..
2. Null and Alternative Hypotheses

Admission Prediction

1. Parameter of interest
   - Y=admission yes/no
   - x_i=‘factors’
   - $Y=\beta_0+\beta_1 x_1+\beta_2 \beta x^2+\beta_3 x_3+\beta_4 x_4$…..
2. Null and Alternative Hypotheses
   - H_0: all $\beta_i = 0$
   - H_A: At least one $\beta_i \neq 0$

Admission Prediction

3. Choose Test Statistic



Admission Prediction

3. Choose Test Statistic

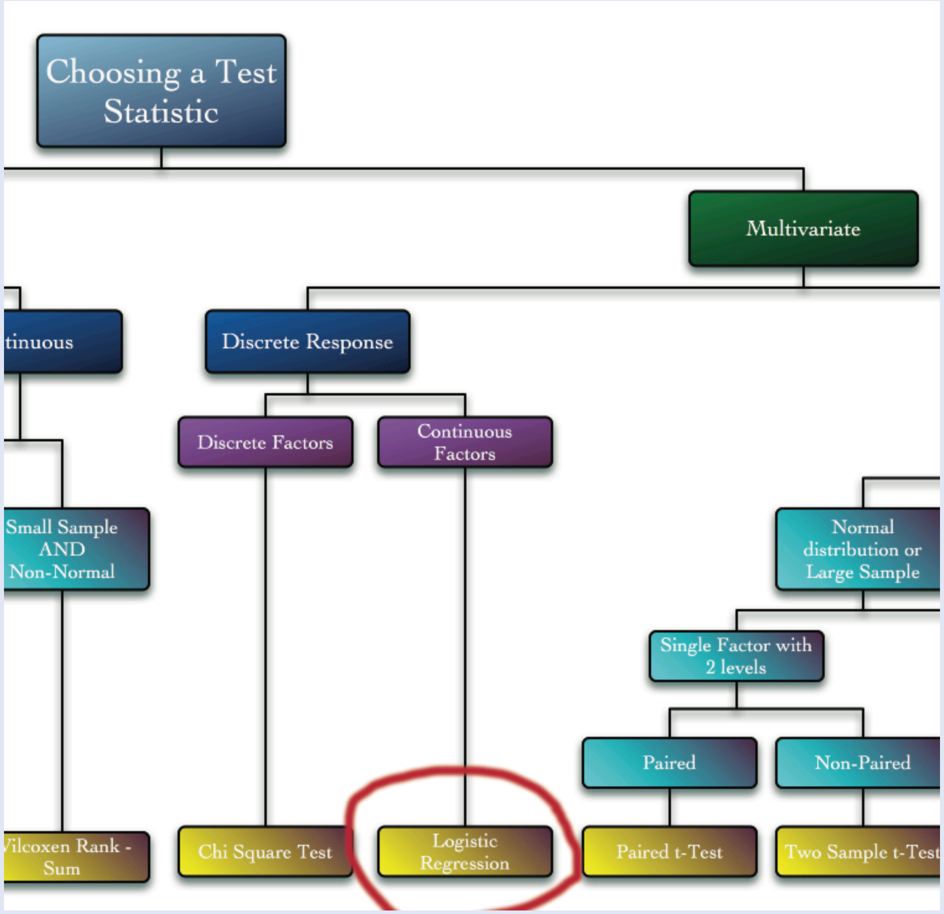
Multivariate

↓

Discrete Response

↓

Continuous Factors



Triage Accuracy

4. Rejection Region

- Use alpha =0.05
- Reject if p<0.006 WHY???

Triage Accuracy

4. Rejection Region

- Use alpha =0.05
- Reject if p<0.006

$$p_i < \frac{\alpha}{m}$$

$$p_i < \frac{0.05}{8}$$

$$p_i < 0.00625$$

Admission Prediction

```
Call:
glm(formula = Admitted ~ Age + Sex + CTAS + Pulse + Resp + GCS +
    Systolic + Ambulance, family = binomial(link = "logit"),
    data = na.omit(admit))

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.8464  -0.6494  -0.3966  -0.2054   2.7613

Coefficients:
             Estimate Std. Error z value  Pr(>|z|)
(Intercept) -1.362313   1.881715  -0.724  0.46908
Age          0.029226   0.005678   5.147 2.65e-07 ***
SexM         0.539982   0.233519   2.312  0.02076 *
CTAS        -0.470511   0.174033  -2.704  0.00686 **
Pulse        0.003543   0.006536   0.542  0.58778
Resp         0.079825   0.032836   2.431  0.01506 *
GCS         -0.133719   0.109911  -1.217  0.22375
Systolic    -0.005160   0.005627  -0.917  0.35915
Ambulance1   1.239053   0.246058   5.036 4.76e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 617.2  on 565  degrees of freedom
Residual deviance: 485.6  on 557  degrees of freedom
AIC: 503.6

Number of Fisher Scoring iterations: 5
```

Admission Prediction

6. Decide if null hypothesis rejected:

- REJECT

Admission Prediction

7. State Conclusion in Context

Need for admission appears to be correlated with age and arrival by ambulance.

DM Curriculum Scores

A researcher is investigating the impact of a new disaster medicine curriculum. The researcher has pre and post-test scores for 12 students.

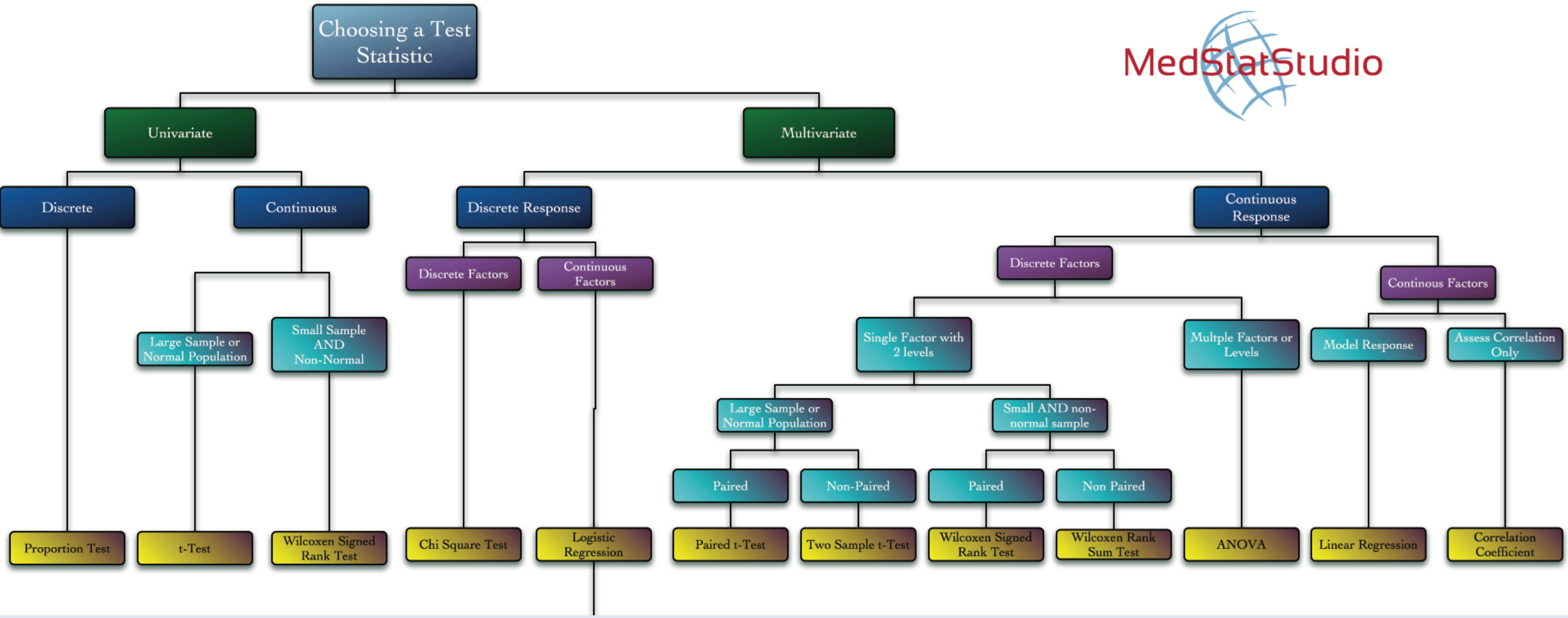
DM Curriculum Scores

1. Parameter of Interest
   - μ_D where D=X-Y
   - (Difference between pre and post test score for each student)
2. Determine Null and Alternative Hypotheses
   - H_0: $\mu_D=0$
   - H_A: $\mu D \neq 0$

4. State Rejection Region
   - $\alpha=0.05$
   - Reject if $p<0.05$

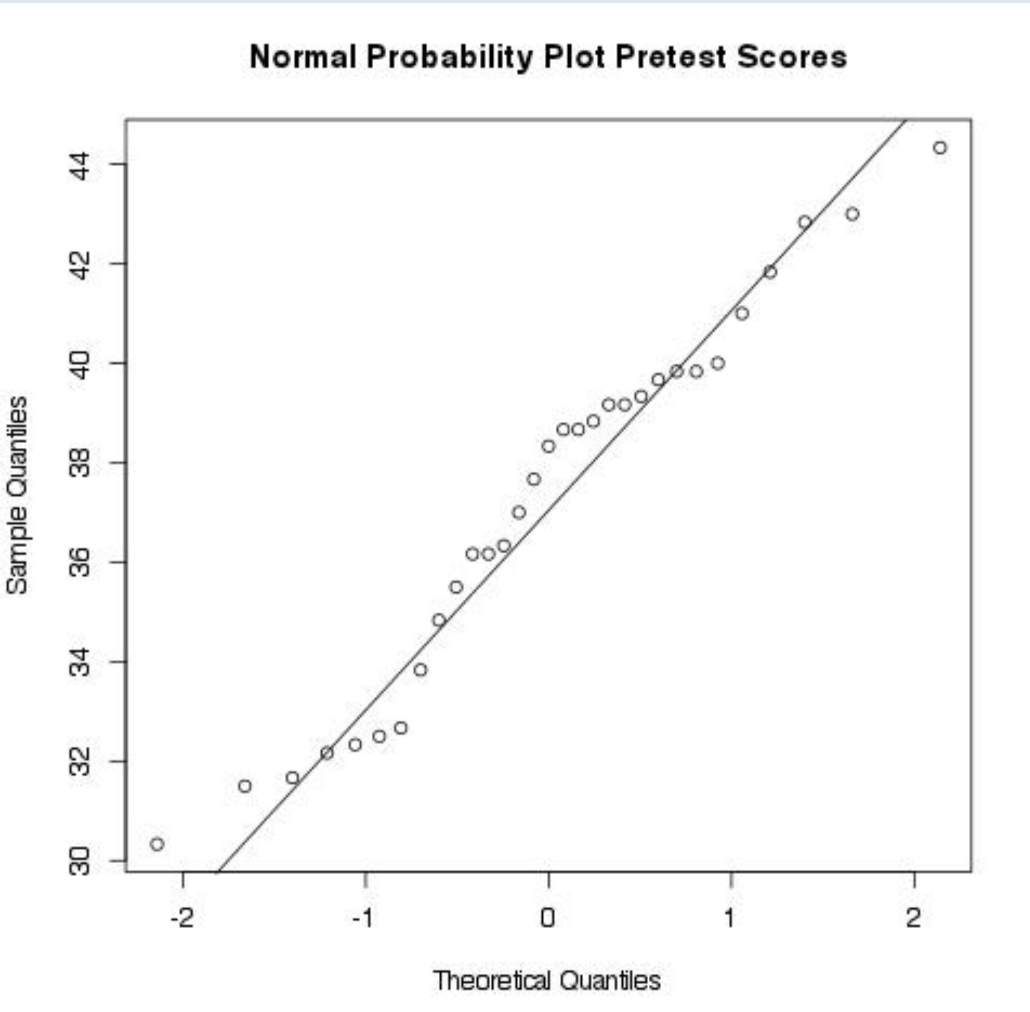
DM Curriculum Scores

3. Define the test statistic



DM Curriculum

Is this normal?



Is this Paired?

We are again interested in testing hypotheses about the difference $\mu_1 - \mu_2$. The denominator of the two-sample t test was obtained by assuming independent samples and applying the rule $V(\overline{X} - \overline{Y}) = V(\overline{X}) + V(\overline{Y})$. However, with paired data, the X and Y observations within each pair are often not independent, so $\overline{X}$ and $\overline{Y}$ are not independent of one another. We must therefore abandon the two-sample t test and look for an alternative method of analysis.

DM Curriculum

3. Define the test statistic

Multivariate

↓

Continuous Response

↓

Discrete Factors

↓

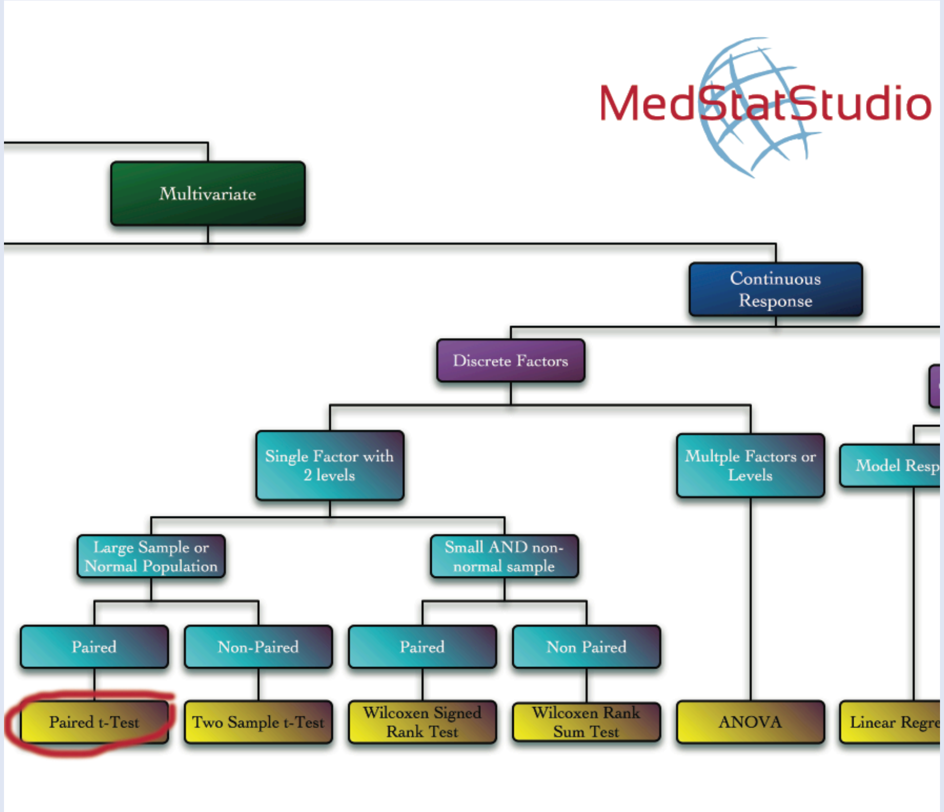
Single factor with 2-levels

↓

Large Sample

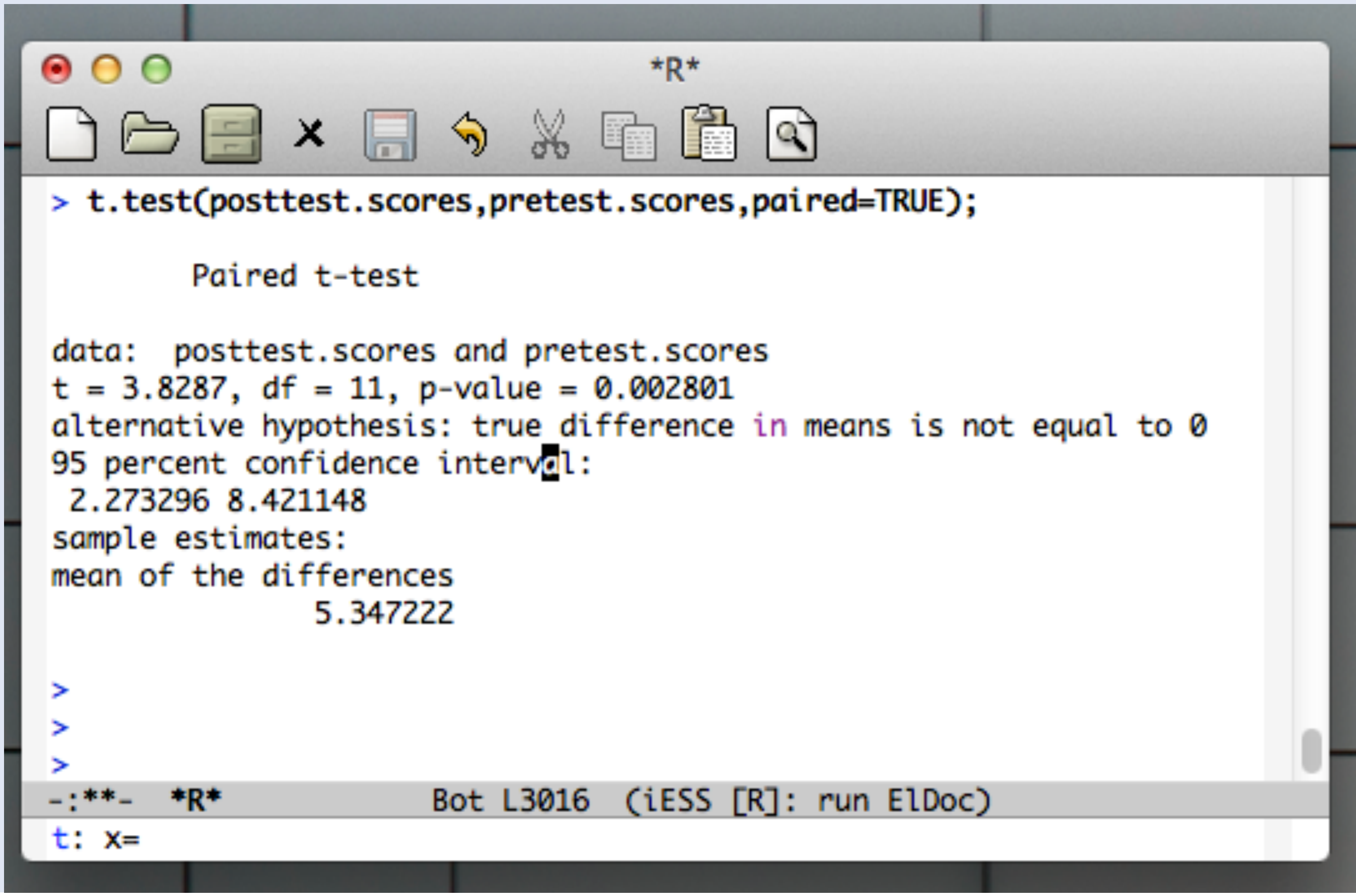
↓

Paired



DM Curriculum Scores

5. Calculate Test Statistic



DM Curriculum Scores

7. State conclusion in context.

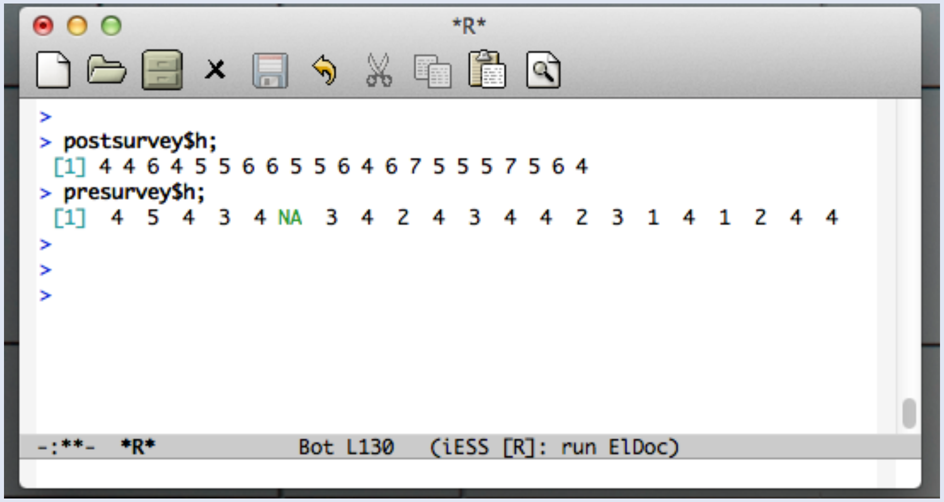
There was a significant improvement in test scores after the curriculum ($p<0.003$). The mean improvement in test score was 5.3 points (95% confidence interval: 2.3 to 8.4).

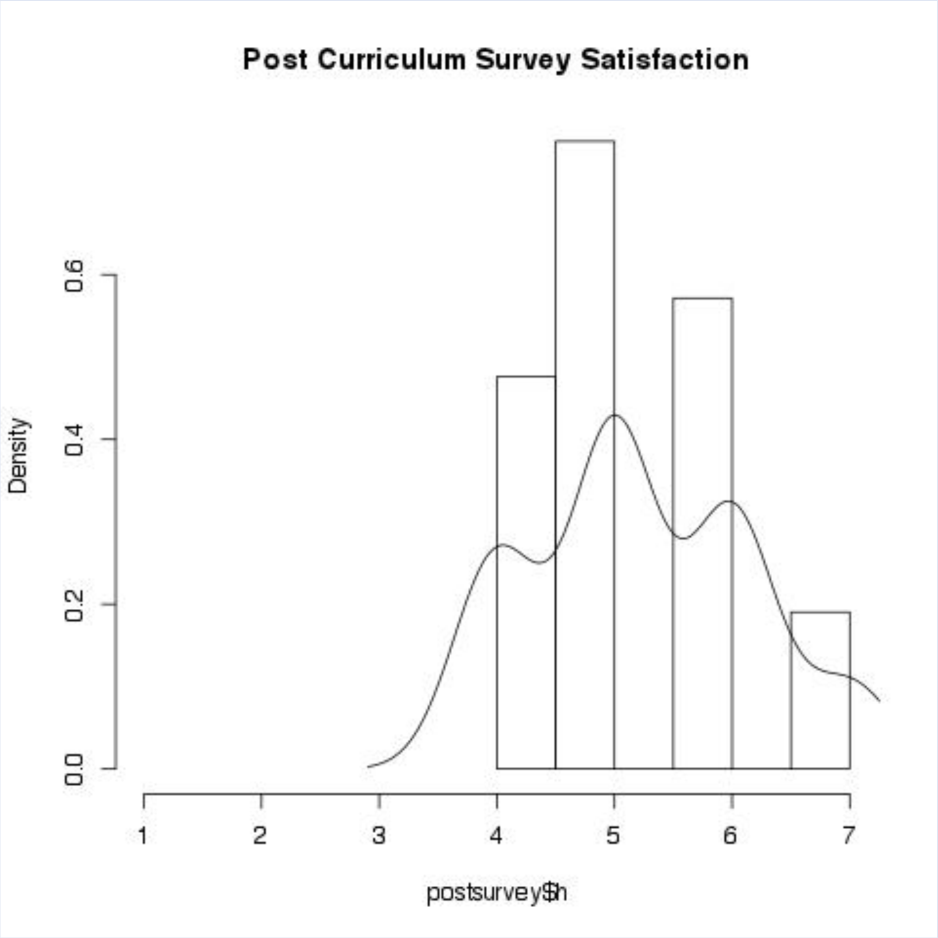
Survey Satisfaction

A researcher measured satisfaction with a new DM curriculum by means of a survey. The survey was administered to third year residents prior to the introduction of the new curriculum and then to the next years third year residents after the curriculum.

The researcher wants to show that the overall satisfaction has increased.

Survey Satisfaction





Survey Satisfaction

1. Parameter of interest
   - ???

Survey Satisfaction

1. Parameter of interest
   - μ_{pre} and μ_{post}

Survey Satisfaction

1. Parameter of interest
   - μ_{pre} and μ_{post}

2. Null and Alternative Hypotheses
   - ??????

Survey Satisfaction

1. Parameter of interest
   - μ_{pre} and μ_{post}

2. Null and Alternative Hypotheses
   - H_0: $\mu_{pre} = \mu_{post}$
   - H_A: $\mu_{pre} \neq \mu_{post}$

Survey Satisfaction

1. Parameter of interest
   - μ_{pre} and μ_{post}

2. Null and Alternative Hypotheses
   - H_0: $\mu_{pre} = \mu_{post}$
   - H_A: $\mu_{pre} \neq \mu_{post}$

3. Test Statistic
   - ??????

Survey Satisfaction

3. Test Statistic

Multivariate

↓

Continuous Response

↓

Single factor 2 levels

↓

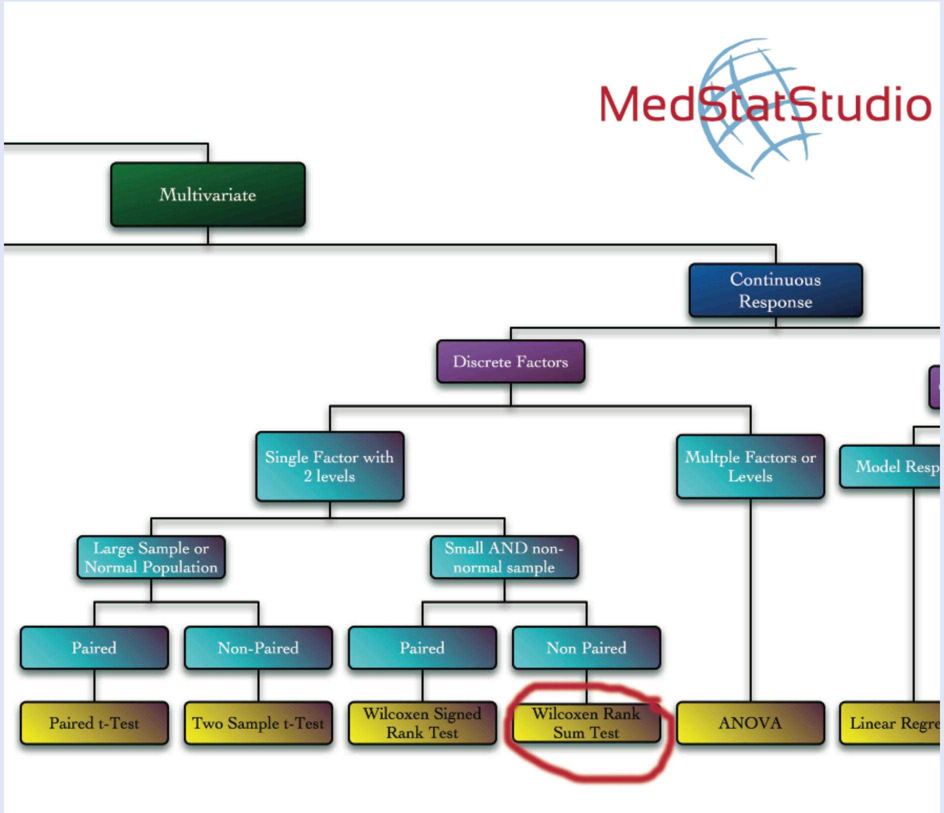
Small and non-normal

↓

Not Paired

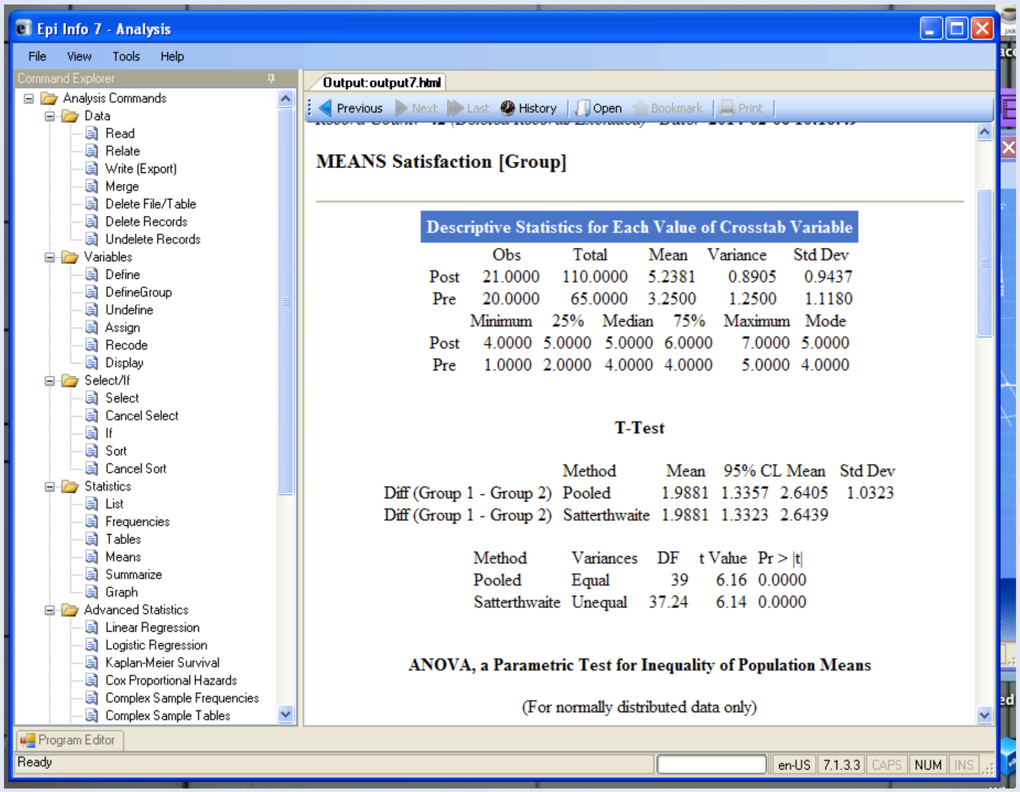
↓

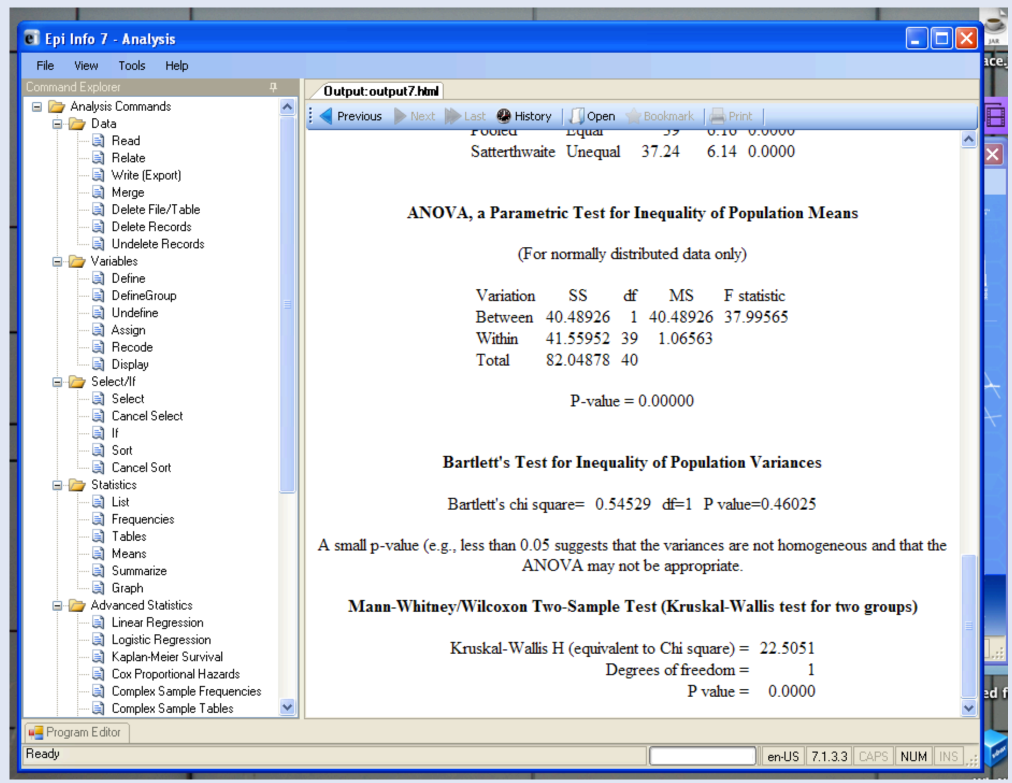
Wilcoxen Rank-sum



Survey Satisfaction

5. Calculate the test statistic





Survey Satisfaction

7. State Conclusion in Context

- Median satisfaction score before the introduction of the curriculum was 4/7. Median satisfaction after the introduction of the curriculum was 5/7. We reject the null hypothesis of no difference, and conclude that resident satisfaction was significantly higher on after the introduction of the curriculum. ($p<0.0001$)

ED Shifts

A researcher suspects that the emergency department shift workload is not equal across all shifts. To investigate this possibility, number of patients seen during each shift was tabulated for each of the 7 daily shifts for 1 week (49 shifts total).

ED Shifts

H_0: All shifts have same volume

H_A: At least one of the shifts has different volume

α=0.05

Test Statistic?

ED Shifts

3. Define Test statistic

Multivariate

↓

Continuous Response

↓

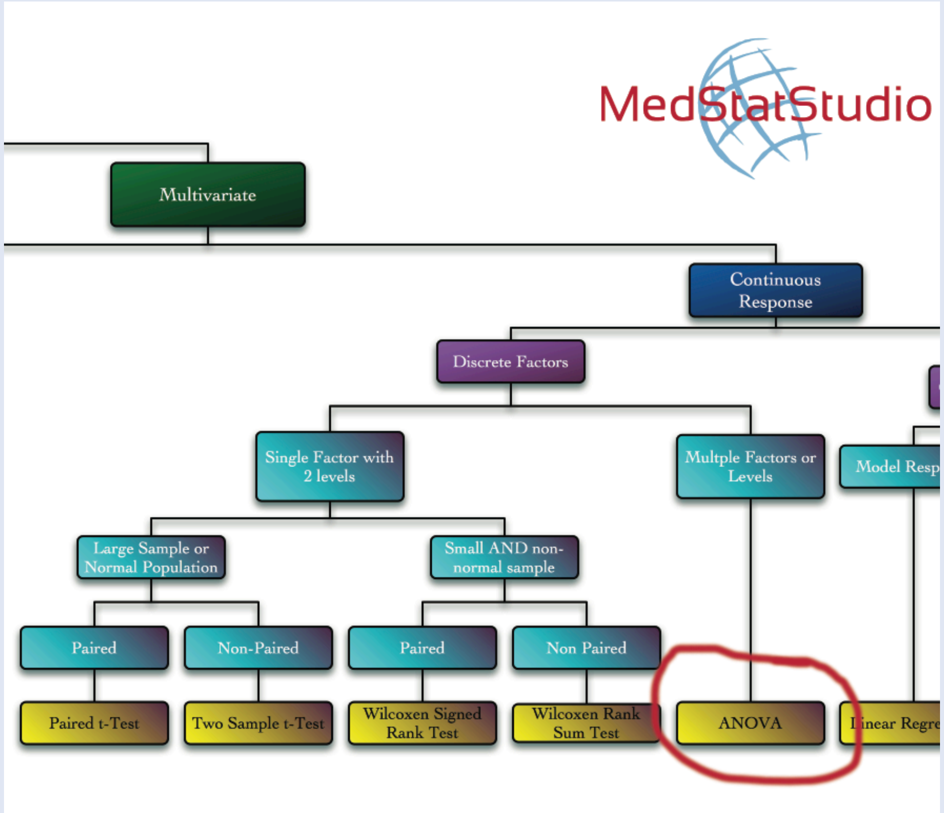
Discrete Factors

↓

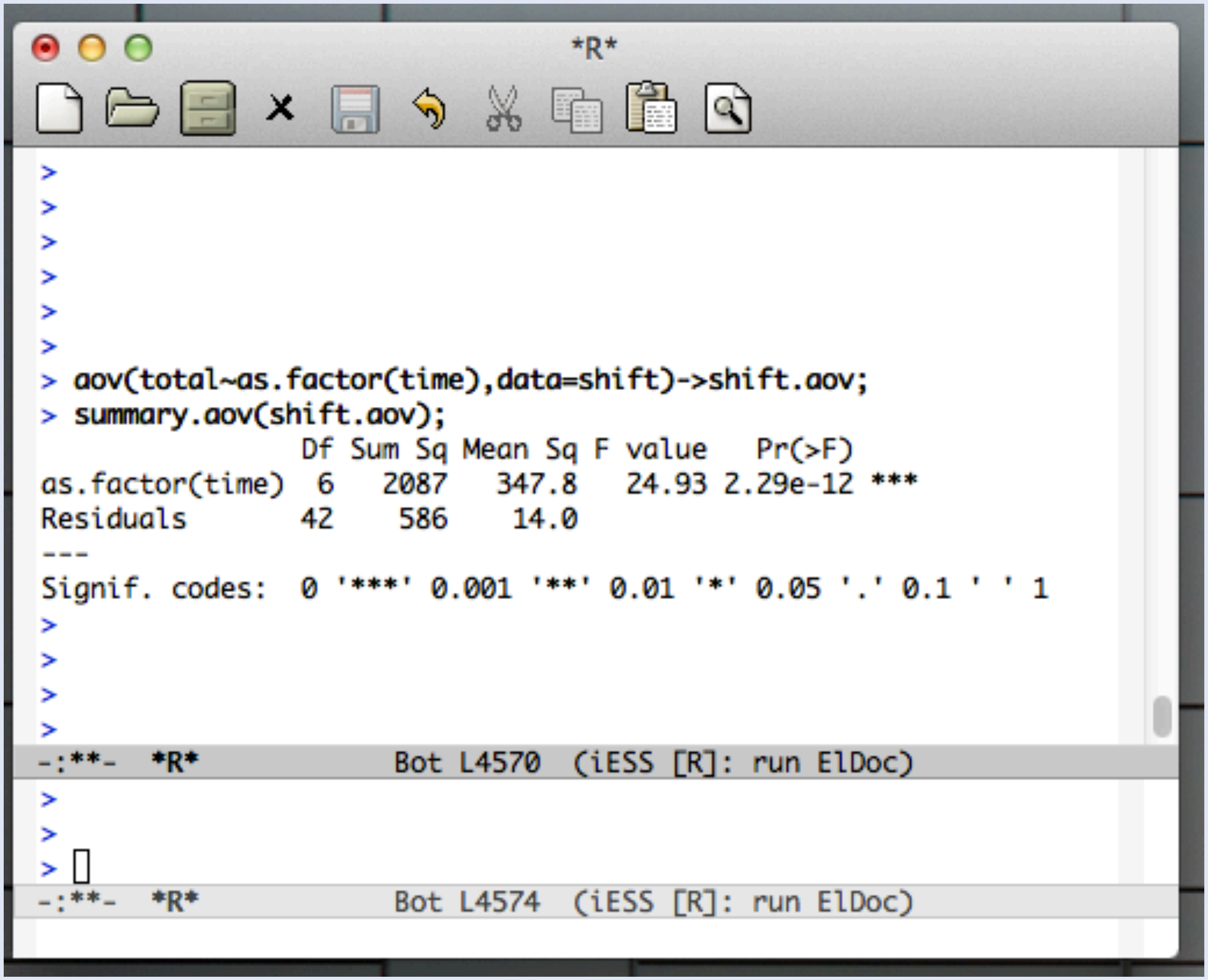
Multiple Levels

↓

ANOVA



ED Shifts


R
> aov(total~as.factor(time),data=shift)->shift.aov;
> summary.aov(shift.aov);
Df Sum Sq Mean Sq F value Pr(>F)
as.factor(time) 6 2087 347.8 24.93 2.29e-12 ***
Residuals 42 586 14.0

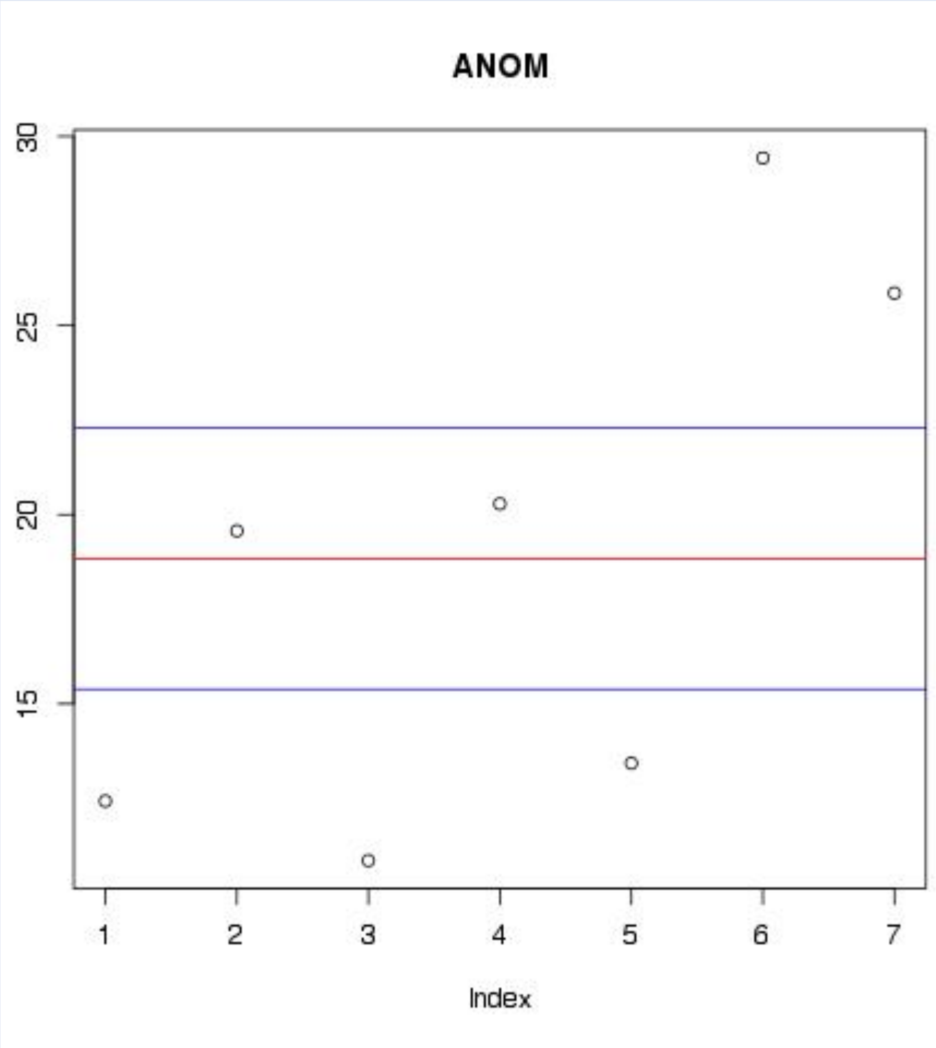
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
-:**- *R* Bot L4570 (iESS [R]: run ElDoc)
-:**- *R* Bot L4574 (iESS [R]: run ElDoc)

ED Shifts

```
*R*
>
>
> TukeyHSD(shift.aov);
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = total ~ as.factor(time), data = shift)

$`as.factor(time)`
              diff         lwr         upr     p adj
s09-s06  7.1428571   0.9623041 13.32341018 0.0143804
s12-s06 -1.5714286  -7.7519816  4.60912446 0.9850119
s14-s06  7.8571429   1.6765898 14.03769589 0.0052639
s18-s06  1.0000000  -5.1805530  7.18055303 0.9987114
s19-s06 17.0000000  10.8194470 23.18055303 0.0000000
s24-s06 13.4285714   7.2480184 19.60912446 0.0000007
s12-s09 -8.7142857 -14.8948387 -2.53373268 0.0014726
s14-s09  0.7142857  -5.4662673  6.89483875 0.9998130
s18-s09 -6.1428571 -12.3234102  0.03769589 0.0523439
s19-s09  9.8571429   3.6765898 16.03769589 0.0002480
s24-s09  6.2857143   0.1051613 12.46626732 0.0439440
s14-s12  9.4285714   3.2480184 15.60912446 0.0004880
s18-s12  2.5714286  -3.6091245  8.75198161 0.8535755
s19-s12 18.5714286  12.3908755 24.75198161 0.0000000
s24-s12 15.0000000   8.8194470 21.18055303 0.0000001
s18-s14 -6.8571429 -13.0376959 -0.67658982 0.0211188
s19-s14  9.1428571   2.9623041 15.32341018 0.0007621
s24-s14  5.5714286  -0.6091245 11.75198161 0.1014928
s19-s18 16.0000000   9.8194470 22.18055303 0.0000000
s24-s18 12.4285714   6.2480184 18.60912446 0.0000038
s24-s19 -3.5714286  -9.7519816  2.60912446 0.5626816

>
-:**-  *R*          Bot L4604   (iESS [R]: run ElDoc)
menu-bar file one-window
```

- ANOVA does not show WHICH levels of the factor are different.
- Need to use another procedure that compares each group to one another

ED Shifts



Sometimes graphical methods are superior

Shifts #6 and #7 clearly see more patients than the others.

ICED Survey

A researcher has completed a survey for a pilot study of a new command-and-control system for emergency departments. The researcher wonders which component of the tool is most important for customer satisfaction

ICED Survey

Part II

For the following questions, please rate your satisfaction on the following scale

1	2	3	4	5	6	7
^						^
Not at All Satisfied						Very Satisfied

F) ______How satisfied were you overall with the ICED product?

G) ______How satisfied were you with the training session on command-and-control?

H) ______How satisfied were you with the ability to quickly learn ICED?

I) ______How satisfied were you with the ease of implementation of ICED during the simulation?

J) ______How satisfied were you with the ICED introductory text?

K) ______How satisfied were you with the ICED organization chart?

L) ______How satisfied were you with the ICED color coding?

M) ______How satisfied were you with the ICED job action sheets?

N) ______How satisfied were you with the ICED forms?

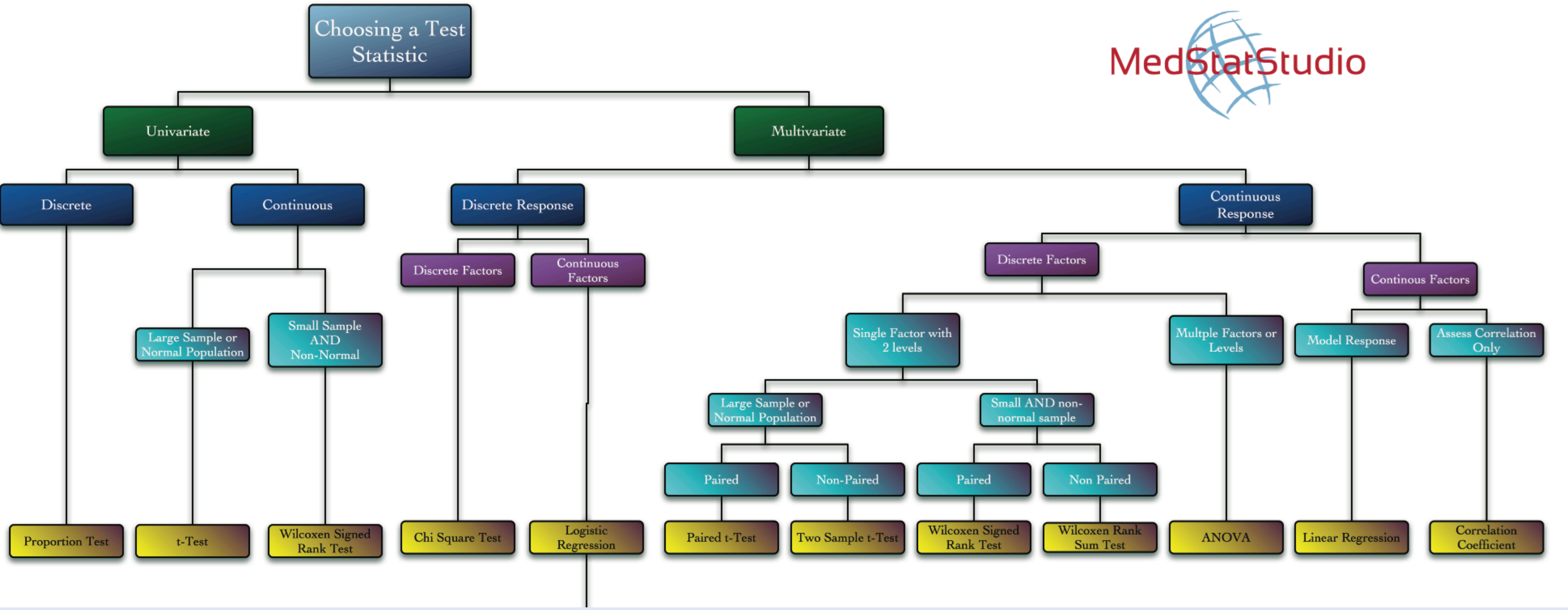
ICED Survey

2. Null and Alternative Hypothesis

- H_0: Overall satisfaction is unrelated to the 5 other components
- H_A: Overall satisfaction is related to at least one of the 5 components.

ICED Survey

3. Define a test statistic



ICED Survey

3. Define a test statistic

Multivariate

↓

Continuous Response

↓

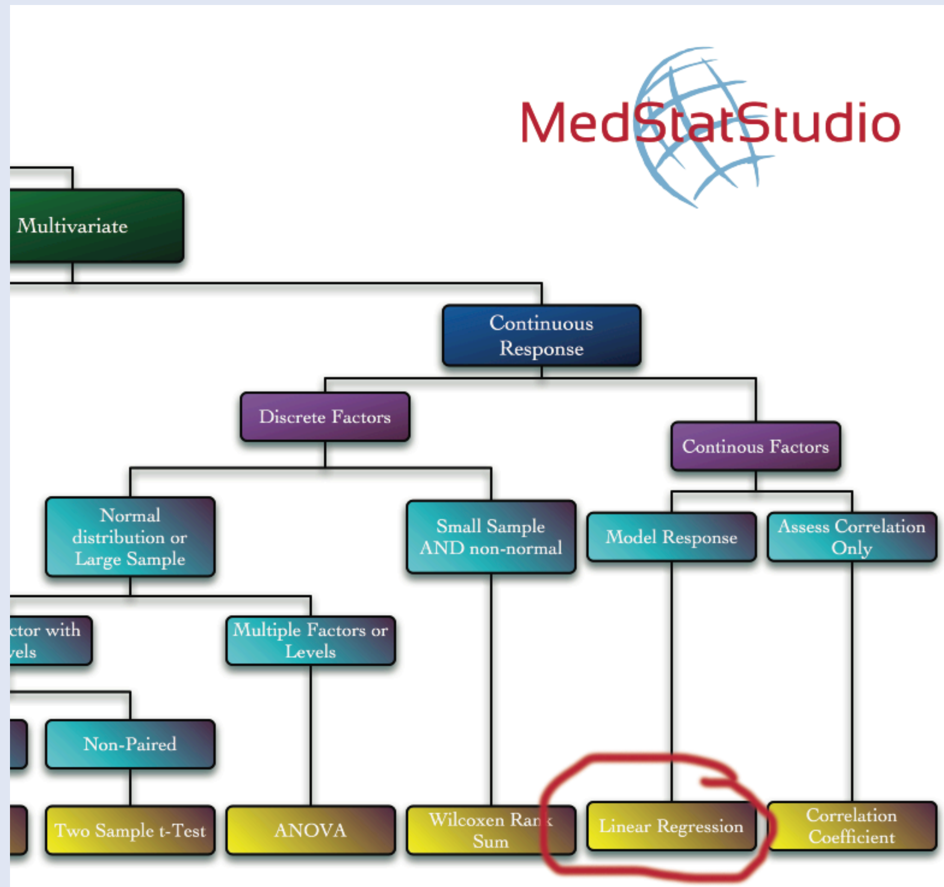
Continuous Response

↓

Model Response

↓

Linear Regression



ICED Survey

5. Calculate Test Statistic

```
*R*

>
> iced.lm<-lm(F~J+K+L+M+N,data=iced.df);
> summary(iced.lm);

Call:
lm(formula = F ~ J + K + L + M + N, data = iced.df)

Residuals:
     Min       1Q   Median       3Q      Max
-2.32717 -0.34352  0.01455  0.46384  1.99984

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.61909    0.32141   1.926  0.05707 .
J            0.53599    0.10643   5.036 2.25e-06 ***
K            0.09327    0.10902   0.856  0.39441
L            0.25387    0.09437   2.690  0.00844 **
M           -0.14550    0.11003  -1.322  0.18923
N            0.16768    0.13207   1.270  0.20733
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.74 on 95 degrees of freedom
  (2 observations deleted due to missingness)
Multiple R-squared: 0.7412,     Adjusted R-squared: 0.7276
F-statistic: 54.42 on 5 and 95 DF,  p-value: < 2.2e-16

>
-:**-  *R*          Bot L5513  (iESS [R]: run ElDoc)
```

ICED Survey

6. REJECT
7. State Conclusion in Context

Overall satisfaction with the ICED tool appears to be most correlated with satisfaction with the introductory text and the color coding.

Pulse and Systolic Pressure

A researcher believes that taking blood pressure at triage during a disaster is unnecessary. She wishes to show that pulse is highly correlated to blood pressure.

She has the Systolic Pressure and Pulse for 2223 emergency department visits

Pulse and Systolic Pressure

1. Parameter of Interest
   - Correlation coefficient ρ

Pulse and Systolic Pressure

2. Determine Null and Alternative Hypotheses

H_0: $\rho=0$ (Population Correlation is zero)

H_A: $\rho\neq0$

Pulse and Systolic Pressure

3. Define Test Statistic

Multivariate

↓

Continuous Response

↓

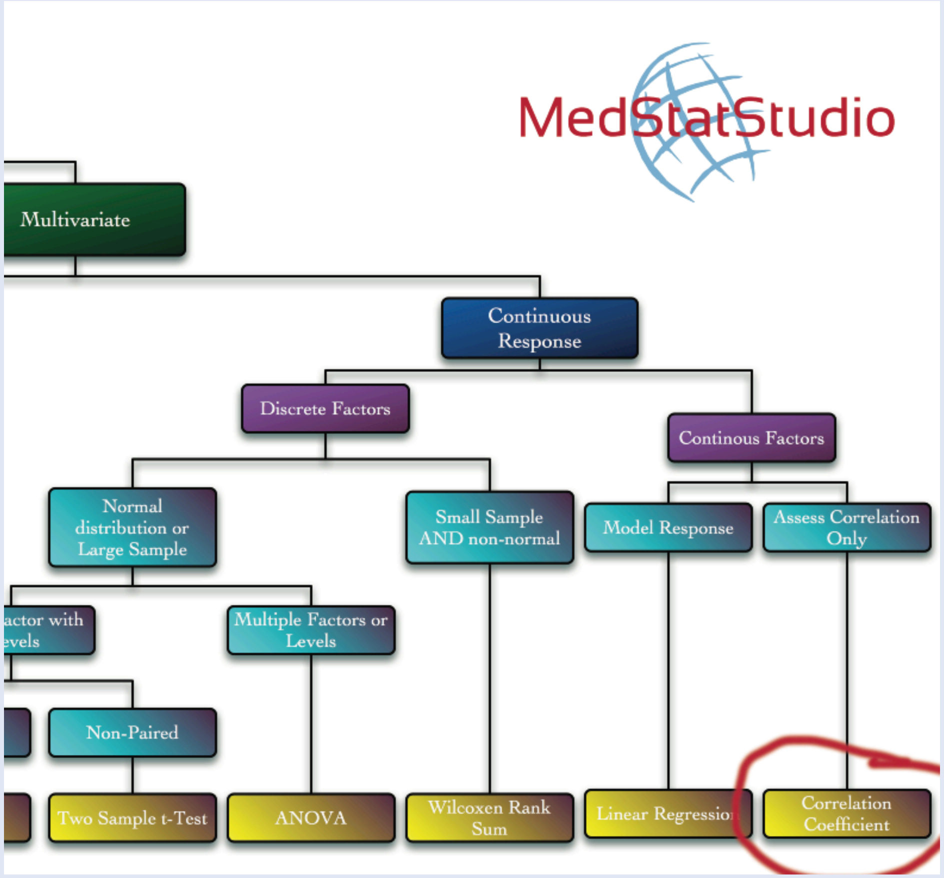
Continous Factors

↓

Correlation only

↓

Correlation Coefficient



Pulse and Systolic Pressure

The sample correlation coefficient: r

Note: $-1 \le r \le 1$

-1 is perfect negative correlation

0 is no linear correlation

1 is perfect positive correlation

Pulse and Systolic Pressure

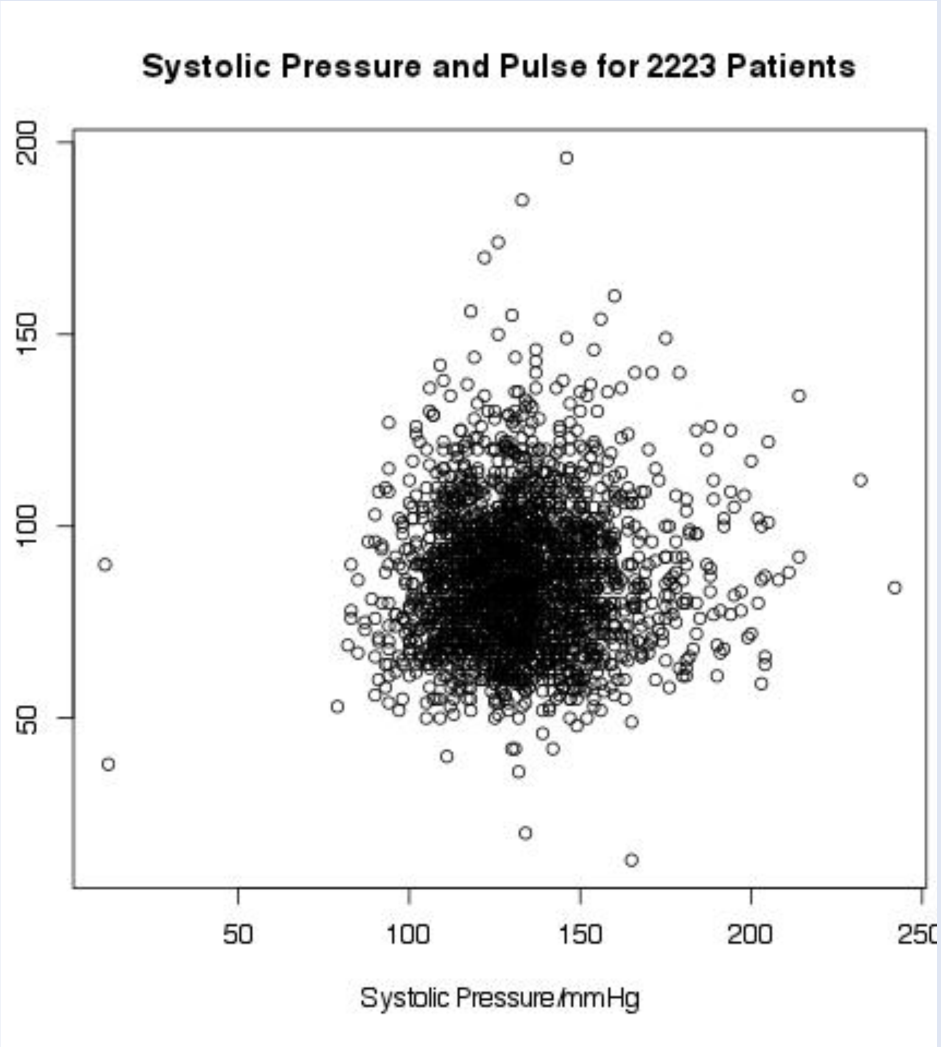
```
>
>
>
>
>
> cor.test(app.df$Systolic,app.df$Pulse);

        Pearson's product-moment correlation

data:  app.df$Systolic and app.df$Pulse
t = 2.4954, df = 2221, p-value = 0.01265
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.01132735 0.09424316
sample estimates:
       cor
0.05287639
```

Pulse and Systolic Pressure

6. Decide if H0 will be rejected:

Reject ???

Pulse and Systolic Pressure



Always Plot !!!

Pulse and Systolic Pressure

7. State conclusion in context

The sample correlation coefficient r was 0.05 (95% confidence interval: 0.01 to 0.09), and thus the null hypothesis of no linear relationship is rejected. ($p<0.02$).

However, the magnitude of the linear correlation is minimal, and the ability to use pulse rate as an alternative to systolic blood pressure for triage purposes is suspect, and would require further study.

Objectives

- Demonstrate the use of an appropriate statistical test for multivariate hypothesis testing including Chi Square test, t-tests, and the correlation coefficient.
- Understand the role of logistic regression, ANOVA, and linear regression.

Quiz Answers: #1

___1. In a study of 14 students, seven were randomized to a disaster medicine training session and seven received only a reference textbook. Scores on a standardized disaster medicine exam given 6 months later appear below.

Training		25	25	50	75	100	100	100
Text Only		75	75	75	100	100	100	100

Quiz Answers: #2

__F_1. In a study of 14 students, seven were randomized to a disaster medicine training session and seven received only a reference textbook. Scores on a standardized disaster medicine exam given 6 months later appear below.

Training		25	25	50	75	100	100	100
Text Only		75	75	75	100	100	100	100

Quiz Answer: #2

___2. A researcher is studying the influence of patients gender on correct disaster triage. Of 100 males, 84 were triaged correctly. Of 130 females, only 65 were triaged correctly.

Quiz Answer: #2

_A_2. A researcher is studying the influence of patients gender on correct disaster triage. Of 100 males, 84 were triaged correctly. Of 130 females, only 65 were triaged correctly.

Quiz Answer: #3

___3. To study anxiety among first responders, 30 volunteers had their heart rates measured 10 minutes before and the again 10 minutes after a simulated disaster scenario.

Quiz Answer: #3

_C_3. To study anxiety among first responders, 30 volunteers had their heart rates measured 10 minutes before and the again 10 minutes after a simulated disaster scenario.

Quiz Answer: #4

___4. A researcher is attempting to develop a survival model for victims of a recent blast injury. The researcher wishes to predict survival versus no survival based on factors such as age, weight, gender, and proximity to the blast in the 120 victims.

Quiz Answer: #4

_B_4. A researcher is attempting to develop a survival model for victims of a recent blast injury. The researcher wishes to predict survival versus no survival based on factors such as age, weight, gender, and proximity to the blast in the 120 victims.

Quiz Answer: #5

___5. A researcher suspects that there is a strong relationship between blood carbon monoxide levels in victims of a building fire and the victims age. Data is available for 21 victims.

Quiz Answer: #5

_H_5. A researcher suspects that there is a strong relationship between blood carbon monoxide levels in victims of a building fire and the victims age. Data is available for 21 victims.

Math Lesson

How to calculate a chi-square test